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MATHEMATICAL MODEL FOR A JET ENGINE WITH ANTI-STALL AUTOMATIC VALVE AIDED COMPRESSOR

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Abstract: *The paper proposes a possible mathematical model for a jet engine, which operates together with its anti-stall valve system. The author has identified how the blow-off valve operating influences the engine's mathematical model. Furthermore, one has studied engine's behavior when its anti-stall valve is driven by a pneumatic automatic system and some simulations were also performed, concerning main engine's output parameter(s) time behavior. Results could be used for similar systems' studying, as well as for further studies of embedded engine control systems.*

Keywords: *compressor, stall, BOV, control, command pressure, mathematical model, speed.*

1. INTRODUCTION

Jet engines for aircraft are built in a large range of performances and types (such as single- or double-spool, single jet or twin jets, with or without afterburning). For a high level thrust engine, there are necessary high compressors pressure ratios π_c^*, high combustor burned gas temperatures T_3^*, as well as alternative thrust augmentation methods (such as afterburning, compressor or combustor water injection).

The more important jet engine is and the bigger its compressor pressure ratio π_c^* is, the more sensitive the compressor is, its sensitivity being represented by the possibility of stall.

Compressor's stall occurs when there is a disruption to the flow of air in the engine compressor, or when the pressure of air entering the engine drops below the pressure in the compressor or air within the compressor drops momentarily as a result of stalling air (disruption in air pressure). Consequently, the compressed air expands and travels quite fast toward the area of less pressure (toward the front and out the back) and it can be explosion-like.

An engine surge occurs when the compressor completely loses compression, phenomena which are more prominent in older jet engines.

Fortunately, nowadays compressor stalls or surges are extremely rare, but when present, they can cause harm to the engine or plane.

Compressor stall often will correct itself as soon as the flow of air in the engine is restored. Often times, the corrective action is given by the pilots by reducing engine power till the engine stabilizes.

Nowadays, following better compressor's gas-dynamic understanding ad with the use of

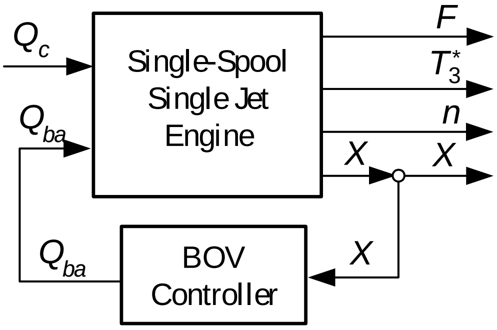

Fig. 1. Embedded system (engine + BOV diagram)

improved modern CAD design methods, this problem has virtually vanished.

One of the stall and surge prevention method for the high-pressure ratio compressors is the blow-off valve (BOV, also known as anti-stall valve) using. An automatic system for the BOV drive is presented in [15].

This paper aim is to introduce and study a possible mathematical model for a jet engine, which operates together with its BOV (as the block diagram in fig. 1 shows), when its working line intersects the surge line on the compressor's characteristic map.

2. SINGLE JET ENGINE MATHEMATICAL MODEL

2.1. Jet engine as controlled object. Aircraft jet engines as controlled objects are studied in [13] and [14], where the authors have identified, amongst a multitude of parameters, possible control parameters (inputs), controlled parameters (outputs), as well as theoretical command laws.

As input parameters, one has identified only three: a) the combustor fuel flow rate Q_c (for all engine types); b) the exhaust nozzle's opening A_5 (only for engines with variable-area nozzles); c) the afterburner fuel flow rate Q_p (only for the engines with afterburning). Other input parameters may be identified when the engine has new facilities/parts with special destination, which could introduce new control loops (see fig. 1), where the blew-off air mass flow Q_{ba} is such an input. Meanwhile, BOV-controller uses as input another engine's output parameter (formally identified as *X*).

Single spool jet engine's most important outputs (controlled parameters) are: spool (shaft) speed (also called engine speed) *n* and combustor hot gases temperature T_3^*. As inputs (control parameters), one may use the fuel flow rate Q_c and, if the engine has a variable exhaust nozzle, its gases exhaust area A_5. Possible mixed control schemes, using the above-mentioned parameters, are presented in [12], [13] and [14].

Engine's model, as presented in [13] and [14] by a matrix description is

$$[A]\times(u)=(b), \tag{1}$$

where $[A]$ is engine matrix, $(u)-$ output parameters vector and $(b)-$ input parameters vector, as follows:

$$A=\begin{bmatrix} T_1 s+\rho_1 & -k_{1T3} & 0 & -k_{1p2} & k_{1p4} \\ k_{2n} & -k_{2T3} & 0 & k_{2p2} & 0 \\ 0 & -1 & 1 & -k_{3p2} & -k_{3p4} \\ 0 & k_{4T3} & k_{4T4} & k_{4p2} & k_{4p4} \\ k_{5n} & k_{5T3} & 0 & k_{5p2} & 0 \end{bmatrix} \tag{2}$$

$$u^T=\left(\overline{n} \quad \overline{T_3^*} \quad \overline{T_4^*} \quad \overline{p_2^*} \quad \overline{p_4^*}\right). \tag{3}$$

$$b^T=\left(0 \quad 0 \quad 0 \quad 0 \quad k_{5Qc}\overline{Q}_c\right). \tag{4}$$

The involved co-efficient are used with their expressions described in [13].

Engine's main output parameters are $\overline{n}$ and $\overline{T_3^*}$; their expressions should be issued by solving system (1) using the Cramer method:

$$X=\frac{det(A_X)}{det(A)}, \tag{5}$$

where *X* is the generic annotation for each one of the output parameters vector's elements, $det(A)-$ determinant of matrix *A*, $det(A_X)-$ determinant of A_X-matrix, A_X- matrix corresponding to *X*-parameter, obtained by the appropriate replacing of its column with input (b) column vector. The presence of a new loop, as in fig.1, modifies the model, by introducing a new input parameter, which is the evacuated air mass flow Q_{ba}.

2.2. Updated model. Because of the BOV operation, an air mass flow Q_{ba} is evacuated from the compressor's air stream, which modifies engine's mass flow equation. So, the mass flow rate balance equation becomes

$$Q_a - Q_{ba} = Q_m, \tag{6}$$

where Q_a is compressor mass air flow rate and Q_m – combustor air flow rate (air involved in the fuel's combustion).

Consequently, one obtains $Q_{ba} = Q_a - Q_m$ which can be also written, using linear deviations, as

$$\Delta Q_{ba} = \Delta Q_a - \Delta Q_m, \tag{7}$$

or else

$$\Delta Q_{ba} = \left(\frac{\partial Q_a}{\partial p_2^*}\right)_0 \Delta p_2^* + \left(\frac{\partial Q_a}{\partial n}\right)_0 \Delta n - \\ - \left(\frac{\partial Q_m}{\partial p_3^*}\right)_0 \Delta p_3^* - \left(\frac{\partial Q_m}{\partial T_3^*}\right)_0 \Delta T_3^*. \tag{8}$$

Considering that, for a steady state regime, $p_3^* = \sigma_{cb}^* p_2^*$, where σ_{cb}^* – combustor's pressure co-efficient, one obtains

$$\frac{Q_{ba0}}{Q_{a0}} \frac{\Delta Q_{ba}}{Q_{ba0}} = \left(\frac{p_{20}^*}{Q_{a0}}\right)\left(\frac{\partial Q_a}{\partial p_2^*}\right)_0 \frac{\Delta p_2^*}{p_{20}^*} + \left(\frac{n_0}{Q_{a0}}\right) \cdot \\ \cdot \left(\frac{\partial Q_a}{\partial n}\right)_0 \frac{\Delta n}{n_0} - \sigma_{cb}^* \left(\frac{p_{30}^*}{Q_{m0}}\right)\left(\frac{\partial Q_m}{\partial p_3^*}\right)_0 \frac{\Delta p_3^*}{p_{30}^*} - \\ - \left(\frac{T_{30}^*}{Q_{m0}}\right)\left(\frac{\partial Q_m}{\partial T_3^*}\right)_0 \frac{\Delta T_3^*}{T_{30}^*}, \tag{9}$$

which may be presented more simply as

$$k_{2ba}\overline{Q}_{ba} = \left(k_{2pc2} - \sigma_{cb}^* k_{2pt2}\right)\overline{p}_2^* + k_{2n}\overline{n} - \\ - k_{2T3}\overline{T}_3^*. \tag{10}$$

One can observe that the co-efficient k_{2p2} in A-matrix in (2) should be replaced by the co-efficient $\left(k_{2pc2} - \sigma_{cb}^* k_{2pt2}\right)$, which can offer better information about BOV operating. Meanwhile, vector b should be adjusted, by adding the term $k_{2ba}\overline{Q}_{ba}$ as second element.

3. COMPRESSOR STALL

Technically speaking, stall occurs when the engine's working line intersects the surge line on the characteristic map, which means that gas stream through the combustor and/or the turbine is disrupted and the shallow border between surge line and working line is crossed, especially near the idle regime.

Fig. 2 shows a characteristic map of a compressor; formally, if stall occurs, one may consider that engine's working line become AS instead of AB. Point A, corresponding to idle regime, is located very close to the surge line, which makes the stall occurrence a high possibility.

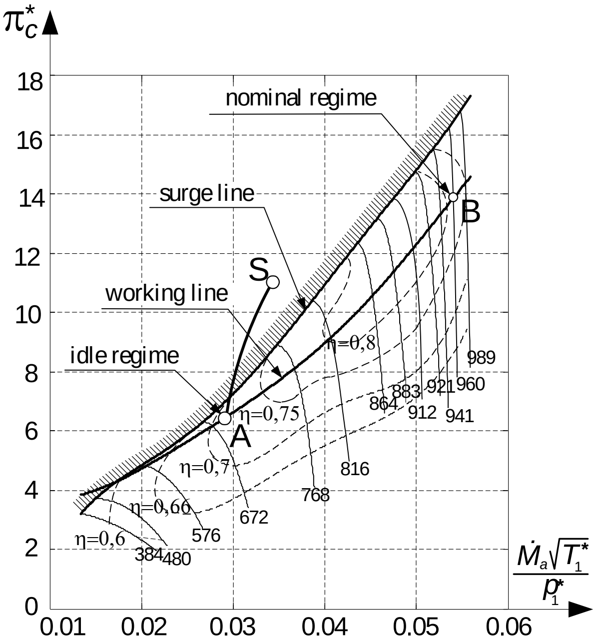

Fig. 2. Compressor characteristic map

In order to introduce the stall into the engine's model, one has to consider the new form of k_{2p2}, $k_{2p2}^{/} = k_{2pc2} - \sigma_{cb}^{*} k_{2pt2}$, where

$$k_{2pc2} = \left(\frac{p_{20}^{*}}{Q_{a0}}\right)\left(\frac{\partial Q_a}{\partial p_2^{*}}\right)_0, \tag{11}$$

$$k_{2pc2} = \left(\frac{p_{30}^{*}}{Q_{m0}}\right)\left(\frac{\partial Q_m}{\partial p_3^{*}}\right)_0. \tag{12}$$

First co-efficient should respect engine's normal operation, which means that the curve where the slope $\left(\frac{\partial Q_a}{\partial p_2^{*}}\right)_0$ is calculated will be the working line AB. Meanwhile, in order to simulate the flow disruption, one has to calculate the second co-efficient, in (12), using for the term $\left(\frac{\partial Q_m}{\partial p_3^{*}}\right)_0$ the AS-curve's slope value (crossing the surge line).

It leads to an important modifying of k_{2p2} value and, together with the presence of $k_{2ba}\overline{Q}_{ba}$ between the elements of *b*-vector, to a new form of engine's mathematical model.

4. EMBEDDED SYSTEM'S MODEL

Jet engine operating with its BOV may be identified as controlled object by its new model, which consists of the updated jet engine model and the BOV model, as follows.

4.1. Jet engine updated model. It contains the new input $\overline{Q}_{ba}$, as well as the new form of the co-efficient k_{2p2}, so the fourth element in the second line in matrix *A* (formula (2) in section 2) becomes $k_{2p2}^{/}$ and the matrix becomes $A^{/}$; output vector remains the same, but the input vector b^{T} becomes

$$b^{/T} = \begin{pmatrix} 0 & -k_{2ba}\overline{Q}_{ba} & 0 & 0 & k_{5Qc}\overline{Q}_c \end{pmatrix}. \tag{13}$$

One can observe that air pressure behind the compressor p_2^{*} is located amongst the engine secondary output parameters, so it becomes the inner feedback of the embedded system (see parameter *X* in fig. 1).

The new system

$$\left[A^{/}\right] \times (u) = \left(b^{/}\right) \tag{14}$$

shall be solved using Cramer-methods and one obtains for engine's speed parameter $\overline{n}$ and for the compressor's exhaust pressure $\overline{p}_2^{*}$ parameter new expressions

$$\overline{n} = \frac{k_{cc}\,\overline{Q}_c + k_{cb}\,\overline{Q}_{ba}}{\tau_m\, s + \rho_m}, \tag{15}$$

$$\overline{p}_2^{*} = \frac{\left(\tau_{cp}\, s + \rho_{cp}\right)\overline{Q}_c + \left(\tau_{pp}\, s + \rho_{pp}\right)\overline{Q}_{ba}}{\tau_m\, s + \rho_m}, \tag{16}$$

where co-efficient involved in eqs. (15) and (16) will be calculated using the new values for $k_{2p2}^{/}$ and k_{2ba}.

The parameters' expressions for temperature T_3^{*} and thrust *F* have similar expressions (see [11], [13] and [14]) and may be calculated (and simulated) if necessary.

4.2. BOV-model. The anti-stall valve's model consists of the blew-off air mass flow equation, as well as of the BOV-controller equation, as follows:

a) Q_{ba} mass flow equation

$$\overline{Q}_{ba} = k_{by}\overline{y} + k_{bp}\overline{p}_2^{*}, \tag{17}$$

where $k_{by} = \frac{\mu_b \sqrt{2}\pi d_b p_{20}^{*} y_0}{Q_{b0}\sqrt{\rho}}$ and

$$k_{bp} = \frac{\mu_b \pi d_b p_{20}^{*} y_0}{Q_{b0}\sqrt{2\rho\left(p_{20}^{*} - p_{H0}\right)}}; \tag{18}$$

b) BOV-controller equation, as determined in [15], with its annotations

$$\overline{y} = \frac{a}{b_2 s^2 + b_1 s + b_0}\overline{p}_2^{*}, \tag{19}$$

$$a = K_{yC}K_{p2}, b_2 = \tau_{CA}T_{\xi}, b_0 = \left(1 - K_D\right),$$
$$b_1 = \left(1 - K_D\right)T_{\xi} - K_{yC}\tau_{Cy} + \tau_{CA}. \tag{20}$$

Consequently, BOV mathematical model may be expressed as

$$\overline{Q}_{ba} = \left(\frac{k_{by}a}{b_2 s^2 + b_1 s + b_0} + k_{bp}\right)\overline{p}_2^{*}. \tag{21}$$

Embedded system operational block diagram is depicted in fig. 3, while system block diagram with transfer functions in fig. 4.

5. SYSTEM'S QUALITY

For a VK-1 type single-spool single-jet engine, *A*-matrix elements values are calculated in [11] and [13]. Based on these values, main parameter expression (engine's speed *n*) is

$$\overline{n} = \frac{1.352\,\overline{Q}_c - 0.269\,\overline{Q}_{ba}}{1.059\,s + 0.7102}, \qquad (22)$$

while secondary parameter $\overline{p}_2^*$ expression is

$$\overline{p}_2^* = \frac{(0.84\,s + 3.086)\overline{Q}_c - (0.975\,s + 7.13)\overline{Q}_{ba}}{1.059\,s + 0.7102} \qquad (23)$$

BOV controller equation, which co-efficient are calculated as presented in [15], has the form

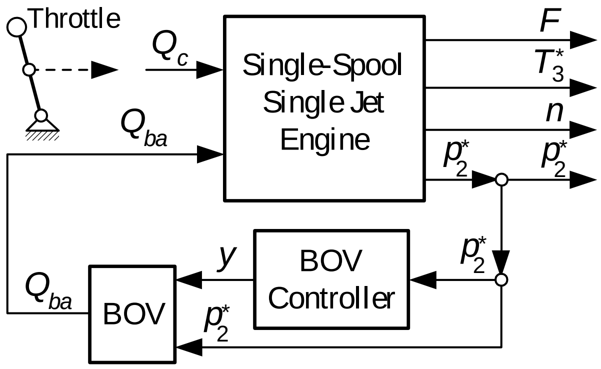

Fig. 3. Embedded system (engine+BOV) operational block diagram

$$\overline{y} = \frac{0.6237}{s^2 + 4.821s + 4.7823}\,\overline{p}_2^*. \qquad (24)$$

Based on the block-diagram with transfer functions in fig. 4 and on the above-determined forms for the system parameters, one has performed a simulation, considering as input parameter engine's fuel mass flow rate $\overline{Q}_c$ and as main output parameter engine's

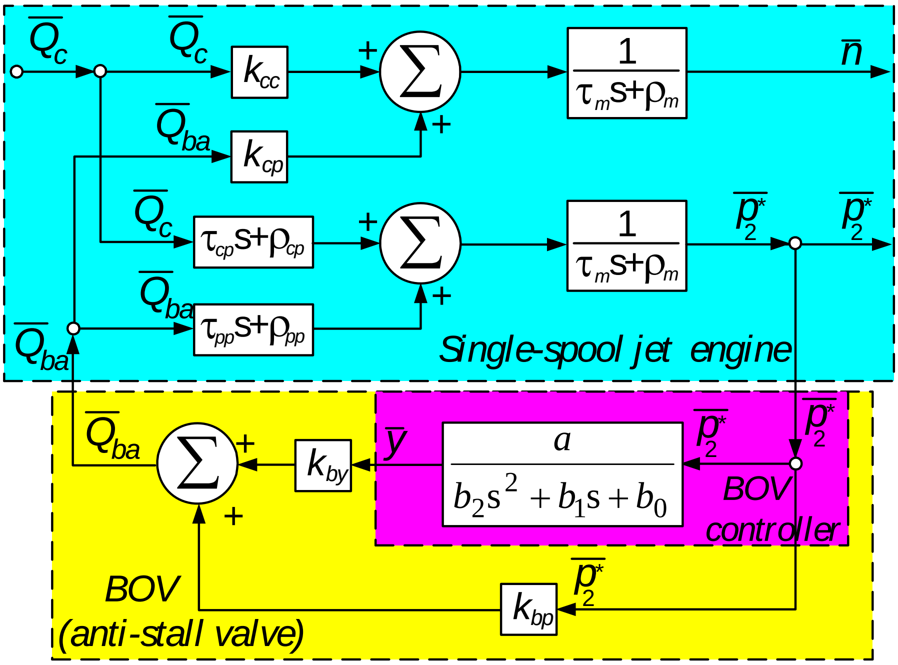

Fig. 4. Embedded system block diagram with transfer functions

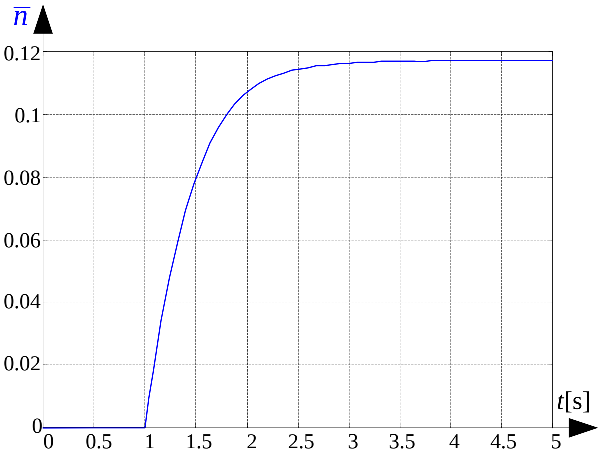

Fig. 5. Embedded system speed step response (fuel mass flow rate step input)

speed $\overline{n}$; meanwhile, one has studied both secondary output parameters (compressor pressure $\overline{p}_2^*$ and BOV opening $\overline{y}$) behavior.

One has considered a step input of the fuel mass flow rate parameter, so output parameters' behavior ($\overline{n}$, $\overline{p}_2^*$ and $\overline{y}$) is system's step response. System's time response (output parameters' values versus time) lays in figures 5, 6 and 7.

Fig. 5 presents main output parameter's (engine speed) behavior when the anti-stall system is active, while fig. 6 presents a comparison between the speed behavior in this situation (curve I) and the speed behavior when the engine is operating after the AS-working line in fig. 2 (curve II), so it become an unstable object. One can observe that BOV-operating makes the engine a stable object from the speed point of view, its stability being aperiodic.

Secondary parameters' ($\overline{p}_2^*$ and $\overline{y}$) time response is presented in fig. 7. One can observe that both output parameters are stable.

BOV opening parameter $\overline{y}$ is aperiodic, because of the chosen co-efficient of the (24) transfer function, in order to assure a smooth mass air flow evacuation; a periodic BOV opening isn't acceptable, being the reason of stall/surge engine behavior.

Air pressure $\overline{p}_2^*$ parameter's behavior is also stable, but one can observe a small initial override; is doesn't matter from the BOV opening point of view, which has an aperiodic behavior. However, if the override grows, or if the stability becomes periodic, it definitely may initiate compressor's stall-operating, because of the possibility of surge line crossing.

CONCLUSIONS

In this paper one has proposed an approach of the unstable engine behavior, caused by the compressor's stall; meanwhile, one has introduced a possibility of correction, by using an anti-stall BOV for the compressor.

One has introduced a new single-spool single jet engine model, based on several existing mathematical models and on the author experience concerning co-efficient

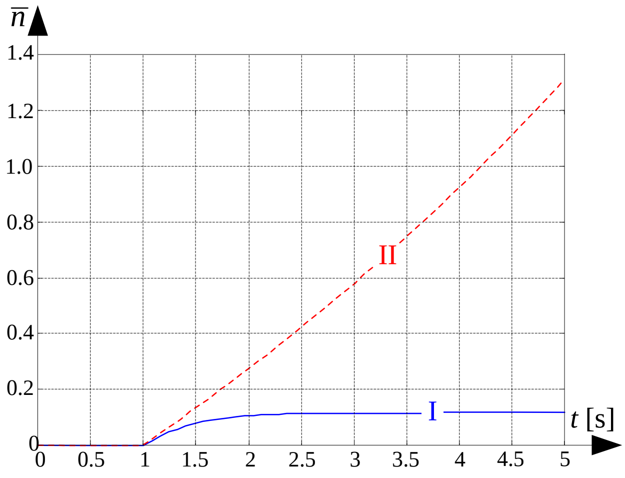

Fig. 6. System speed step response comparison (fuel mass flow rate step input)

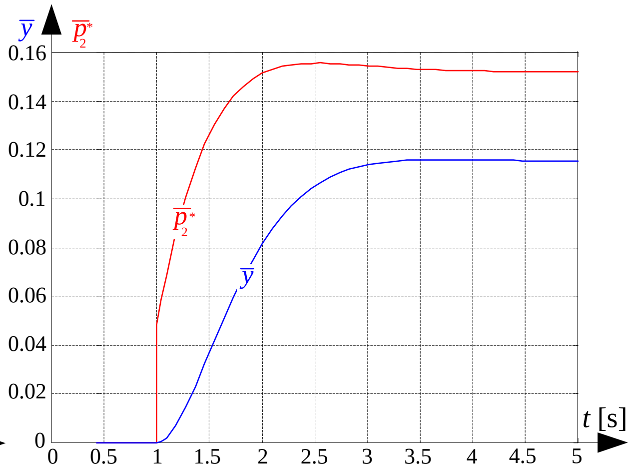

Fig. 7. System step response for secondary output parameters $\overline{p}_2^*$ and $\overline{y}$

calculation for various engine's regimes, different of the cruise regime, as is the idle regime.

BOV presence and operating means another compressor mass airflow balance, because of the evacuated (blew-off) air, which modifies the engine's working line, as well as mathematical model's co-efficient system. Meanwhile, it introduces another input parameter, $\overline{Q}_{ba}$, in addition to the already existing fuel mass flow rate $\overline{Q}_c$. In fact, the new input $\overline{Q}_{ba}$ is a parameter given by the BOV operating and it depends on $\overline{p}_2^*$, the air pressure parameter behind the compressor, which is a secondary engine output parameter; consequently, a new closed loop was added to the engine's model.

Based on another study, concerning a BOV-controller [15], one has built a control scheme for the embedded system (engine+ +BOV+controller), whose mathematical model was determined, as well as its block-diagram with transfer functions.

The above-mentioned elements were used during the simulations; one has considered a step input for the main input parameter $\overline{Q}_c$, while as output parameter one has considered engine's speed (which is the most illustrative parameter, being proportional to the engine thrust), as well as most important secondary output parameters: BOV opening $\overline{y}$ and compressor's pressure $\overline{p}_2^*$ (which is the cause of the studied phenomena and, in the same time, BOV-controller's input parameter).

As fig. 6 shows, when the stall/surge occurs, engine's speed becomes an unstable parameter (curve II). Applying the requested constructive measures (BOV installing) one obtains a stabilization of the engine, its speed parameter becoming aperiodic-stable (curve I), even if its secondary parameter $\overline{p}_2^*$ has a small override and seems to be periodic-stable (as fig. 7 shows). In spite of it, BOV-opening parameter $\overline{y}$ is aperiodic stable (fig. 7) and, consequently, a smooth air evacuation (blow-off) is assured.

Engine's speed parameter's behavior when the BOV-controller is active (fig. 5) shows an appropriate operating, an aperiodic stabilization and an acceptable response time (around 2.5 s), even if its static error is higher than usual (around 12%).

This paper is useful for aircraft engine and control specialists and students. Results could be used for similar systems' studying, as well as for further studies of embedded engine control systems.

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